Normal Systems of Algebraic and Partial Differential Equations

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In our talk we first discuss a joint paper with M. Tonoyan: [On a Multivariate Theory, in "Approximation Theory: A volume dedicated to Blagovest Sendov", (B. Bojanov Ed.), Darba, Sofia, 2002, 212-230]. Here the polynomial interpolation approach is used to introduce the main results on multivariate normal algebraic systems. The characterization of the algebraic systems which have maximal number of distinct and multiple solutions is treated as the first and second versions of multivariate fundamental theorem of algebra for normal systems, respectively. The relation between the second version and a result of B. Mourrain is mentioned.

The main connection between normal algebraic systems and normal systems of PDEs is that the formers serve as the systems of characteristic equations for the latters (with constant coefficients). This multivariate setting possesses all well-known properties of the classic univariate case.

Next we bring a construction which shows that any standard algebraic system, with finite set of solutions, $p(x,y)=0,\ q(x,y)=0$, can be reduced to a normal type algebraic system, namely to the following one of total degree $n+m-1$:

$$\psi_\alpha(x,y)p(x,y)=0, \phi_\beta(x,y)q(x,y)=0,\ \alpha,\beta\in Z_+^2, |\alpha|=n-1, |\beta|=m-1,$$

where n,m are the degrees of p,q, respectively. Here degrees of polynomials ϕ,ψ are $n-1, m-1$ and they have maximal number of zeros satisfying the first and second equations of the initial system, respectively. Beforehand we choose a coordinate system such that the leading terms of p and q have no common (nontrivial) solution. This construction gives some consequences interesting from the algebraic and interpolation points of view.